Computational Logic, Fall 2002	**Handout 18**
TEST 3	**December 10, 2002**

Show all your work. There are two bonus problems. Don't try them before you are done with the core problems: **your grade will be determined by the core problems only!**

Problem 1.

a) Find an **IntH**-derivation of $((A \to B) \to B) \to B \vdash A \to (A \to (A \to B))$. Feel free to use the Deduction Theorem.

b) Internalize this derivation as a combinatory term, possibly using emulated λ-abstraction. Don't bother converting it into a pure combinatory format.

Problem 2. As usual, Church numerals are λ-terms

$\overline{0} = \lambda sz.z$
$\overline{1} = \lambda sz.sz$
$\overline{2} = \lambda sz.s(sz)$
$\overline{3} = \lambda sz.s(s(sz))$
$\overline{4} = \lambda sz.s(s(s(sz)))$
$\ldots$

Consider λ-term $t = \lambda xy.yx$. Find normal forms of the following terms:

a) $t \cdot \overline{2} \cdot \overline{1}$
b) $t \cdot \overline{2} \cdot \overline{n}$
c) $t \cdot \overline{m} \cdot \overline{n}$

Problem 3. Which of the following is provable in **S4**? Provide a justification.

a) $\Box(\Box A \to B) \to (\Box A \to \Box B)$
b) $(\Box A \to \Box B) \to \Box(\Box A \to B)$
c) $\Box(\Box(A \to \Box A) \to A) \to A$

Problem 4.

a) Find an **S4**-derivation of $(\Box A \vee \Box B) \to \Box(\neg\Box A \to B)$
b) Find its realization in **LP**

Problem 5. Given an **S4**-derivation $x{:}A, B \vdash A \wedge B$:

1. $x{:}A$, a hypothesis
2. $x{:}A \to A$, an axiom
3. A, by MP, from 1,2,
4. B a hypothesis
5. $A \to (B \to A \wedge B)$, an axiom
6. $B \to A \wedge B$, by MP, from 3,5
7. $A \wedge B$, by MP, from 4,6,

find a proof polynomial $t(x, y)$ and a derivation in **LP** $x{:}A, y{:}B \vdash t(x, y){:}(A \wedge B)$.

Bonus Problem 1. Let

$? = \lambda abcdefghijklmnopqstuvwxyzr.r(thisisafixedpointcombinator)$;
$\$ = ??????????????????????????$.

Show that \$ is a fixed point combinator, i.e. that $\$F = F(\$F)$ holds for all λ-terms F.

Bonus Problem 2. We know that with respect to Gödel's translation $tr(F) =$ *box each subformula of* F **S4** corresponds to **Int**. Show that **S5** corresponds to the classical logic **Cl** with respect to the same Gödel's translation:

$$\mathbf{Cl} \vdash F \quad \textit{if and only if} \quad \mathbf{S5} \vdash tr(F)$$