Computational Logic, Fall 2002	**Handout 13**
TEST 2	**November 12, 2002**

1	2	3	4	5	Total	Bonus 1	Bonus 2

Full name: ____

The test is open book. Feel free to use any written source. However, you must not communicate with other students during the test!

Statement of integrity: I did not, and will not, break the rules of academic integrity on this exam.

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(Signature)

Show all your work. There are two bonus problems. Don't try them before you are done with the core problems: **your grade will be determined by the core problems only!**

Good luck!

Problem 1.
a) Find an **IntG**-derivation of $\Rightarrow \neg\neg[(A \rightarrow B) \vee (B \rightarrow A)]$
b) Apply the step-by-step algorithm of transforming Gentzen style derivations into **IntN**-derivations from Lecture 5 (cf. also Troelstra and Schwichtenberg, section 3.3.1) to transform a derivation from (a) into a natural derivation of formula $\neg\neg[(A \rightarrow B) \vee (B \rightarrow A)]$.

Problem 2.
a) Find an **IntN**-derivation of $(\neg A \vee B) \rightarrow \neg(A \wedge \neg B)$
b) Apply the step-by-step algorithm of transforming natural derivations into **IntG**-derivations from Lecture 5 (cf. also Troelstra and Schwichtenberg, section 3.3.1) to transform a derivation from (a) into an **IntG** derivation of sequent $\Rightarrow (\neg A \vee B) \rightarrow \neg(A \wedge \neg B)$. Don't simplify the resulting Gentzen style derivation.

Problem 3.
a) Find a natural deduction derivation

$$p \rightarrow q \vdash (p \rightarrow r) \rightarrow (p \rightarrow q \wedge r)$$

b) Find a λ-term corresponding to this derivation.

Problem 4.
a) Find derivation of $A \rightarrow C \vdash A \rightarrow (B \rightarrow C)$ in $_{\rightarrow}$**IntH**.
b) Find internalization of this proof as a combinatory term.

Problem 5. Find the normal forms of the following combinatory terms. Types are suppressed for short, assume they all match properly, x, y, z are variables.

a) $\mathbf{s}(\mathbf{ks})\mathbf{k}x$
b) $\mathbf{s}(\mathbf{ks})\mathbf{k}xy$
c) $\mathbf{s}(\mathbf{ks})\mathbf{k}xyz$
d) $\mathbf{s}(\mathbf{bbs})(\mathbf{kk})xyz$, where $\mathbf{b}$ is $\mathbf{s}(\mathbf{ks})\mathbf{k}$.

Bonus problem 1. Untyped combinatory terms are built from untyped variables $x, y, z, \ldots$ and two untyped constants $\mathbf{k}$ and $\mathbf{s}$. There are two basic reductions copied from the typed case:

$\mathbf{k}uv$ reduces to u
$\mathbf{s}uvw$ reduces to $uw(vw)$.

Show that strong normalization does not hold for untyped combinatory terms.

Bonus Problem 2. Consider a system $\mathbf{IntG}^+$ which is obtained from $\mathbf{IntG}$ by adding new axiom sequents $\Rightarrow \neg\neg A \rightarrow A$ for all A's.

a) Show that $\mathbf{IntG}^+$ is equivalent to the classical logic, i.e. $\mathbf{Cl}$ proves F iff $\mathbf{IntG}^+$ proves $\Rightarrow F$.

b) Show that Cut rule cannot be eliminated in $\mathbf{IntG}^+$. Hint: check the Disjunctive Property for Cut-free derivations in $\mathbf{IntG}^+$.