Computational Logic, Fall 2002	**Handout 7**
TEST 1	**October 8, 2002**

1	2	3	4	5	Total	B1	B2

Full name: ____

The test is open book. Feel free to use any written source. However, you must not communicate with other students during the test!

Statement of integrity: I did not, and will not, break the rules of academic integrity on this exam.

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(Signature)

Show all your work. Each of five core problems is worth 10 points. There are two bonus problems. Don't try them before you are done with the core problems: **your grade will be determined by the core problems only!**

Bonus problems are much harder, and you are welcome to take your time on them. Moreover, if you are thinking of doing some research in the area of Computational Logic, I invite you to try these problems seriously. I will be willing to accept bonus problems solutions from you any time till the beginning of the next lecture on Tuesday October 15.

Good luck!

Problem 1. Prove in intutionistic logic using whatever proof tool you wish (**Int**, **IntG**, etc.). Feel free to use derivations known from lectures, homeworks, practice test solutions, etc., with proper references, there is no need to reproduce those known derivations in your test. However, the solution should be a **derivation**, not just a reference to some completeness theorem which does not produce a specific derivation.

a) $\neg A \to B \vdash \neg\neg(A \vee B)$
b) $(A \vee B) \to C \vdash (A \to C) \wedge (B \to C)$
c) $\vdash \neg\neg(\neg\neg A \to A)$

Problem 2. Show that the following classical tautologies are not provable in intuitionistic logic

a) $(p \to q) \vee (q \to p)$
b) $\neg p \vee \neg\neg p$
c) $[(p \wedge q) \to r] \to [(p \to r) \vee (q \to r)]$

Problem 3. In the classical logic all three propositions $p \to q$, $\neg p \vee q$ and $\neg(p \wedge \neg q)$ are provably equivalent. Please, investigate which of those propositions are equivalent to each other in intuitionistic logic. Order them according to their strength. i.e. show which of six implications ($X \to Y, Y \to X, X \to Z, Z \to X, Y \to Z, Z \to Y$) hold intuitionistically. If the answer is YES, provide a derivation. If the answer is NO, give a countermodel.

Problem 4. Prove that for any propositional formula φ

$$\mathbf{Cl} \vdash \neg\varphi \text{ if and only if } \mathbf{Int} \vdash \neg\varphi$$

Problem 5. Show that $\to$ cannot be expressed in **Int** via other connectives $\wedge, \vee, \bot$. Hints: use the solution of problem 5.8 from the practice kit that for no formula φ without implication **Int** proves φ. Suppose there is a formula $F(p, q)$ without $\to$ such that $\mathbf{Int} \vdash F(p, q) \leftrightarrow (p \to q)$. It is not too difficult now to find a contradiction.

Bonus Problem 1. Show that $\vee$ cannot be expressed in **Int** via other connectives $\wedge, \to, \bot$. Hint: use the solutions of problems 5.3 and 5.4 of the practice kit. Again, suppose there is a formula $F(p, q)$ without $\vee$ such that $\mathbf{Int} \vdash F(p, q) \leftrightarrow (p \vee q)$. Try to cook up a contradiction!

Bonus Problem 2. Some sort of a relevance property for **Int**. If $\mathbf{Int} \vdash A \to B$ and formulas A and B have no propositional variables in common, then $\mathbf{Int} \vdash B$ or $\mathbf{Int} \vdash \neg A$. Hint: prove this for the classical logic first, you will learn a lot.