

CL 2002:

Computational Logic

(Lecture 7)

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This lecture plan

1. Implicational fragment of intuitionistic logic \rightarrow **Int**. Its Hilbert, Gentzen and Natural deduction proof systems.
2. Conservativity of intuitionistic logic with respect to its implicational fragment: for each \rightarrow -only formula F if **Int** proves F then already \rightarrow **Int** proves F . Proof of conservativity via cut-free derivations in **IntG**.
3. Peirce Law $((p \rightarrow q) \rightarrow p) \rightarrow p$ is a classical tautology not derivable in **Int**. Corollary: classical logic **Cl** is not conservative w.r.t. its implicational fragment of (which accidentally coincides with \rightarrow **Int**).
4. Simple types as implication only formulas. Set theoretical (functional) semantics of simple types.
5. Simply typed combinatory terms calculus **CL** \rightarrow a.k.a. simply typed combinatory logic. Combinatory terms as proof terms in \rightarrow **IntH** (Hilbert style proof system for the implicational fragment of intuitionistic logic).
6. Internalization Theorem.: if $A_1, \dots, A_n \vdash B$ in \rightarrow **IntH** then $x_1:A_1, \dots, x_n:A_n \vdash t(x_1, \dots, x_n):B$ in **CL** \rightarrow .
7. Computational meaning of combinators **k** and **s** in set theoretical semantics. Conversions, normal forms of combinatory terms.